APPLICATION OF PERTURBATION THEORY TO TOROIDAL PHASE SHIFTERS

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ABSTRACT

A perturbation treatment provides a quantitative solution for inhomogeneous ferrite waveguides structures. Theory and experiments for the differential phase shift in single and double toroidal phase shifters agree within 5% over a broadband. Treatment of higher order modes, impedances and extension to other devices are outlined.

INTRODUCTION

Exact solutions for double slab ferrite phase shifters (1) have been used to analyze inhomogeneous structures. The invention of the toroidal phase shifter (2) made these idealized solutions obsolete. A new perturbation formalism taking into account the dielectric and magnetic inhomogeneities sequentially yields precise explicit expressions for the differential phase shift of single and double toroidal phase shifters. The formalism has been extended to account for the coupling of higher modes over a broadband.

PERTURBATION PROCEDURE

Starting with the perturbation expression "Microwave Ferrites and Ferrimagnetics" by Lax and Button (3) the actual fields are evaluated by taking into account the depolarizing and demagnetizing factors. The susceptibilities (3) are then expressed as effective quantities and the fields as unperturbed orthonormal LSE and LSM modes of an equivalent dielectrically loaded waveguide. In Figure 1 the unmagnetized ferrite and dielectric insert are treated as a monolithic slab whose effective dielectric constant accounts for the geometry and the appropriate depolarizing factor N as follows:

$$\epsilon^{eff}=\epsilon_f+\frac{\Delta\epsilon}{1+N\frac{\Delta\epsilon}{\epsilon_f}}\,\frac{h-2\delta}{h}\,\frac{2k_\sigma\sigma+\sin 2k_\sigma\sigma}{2k_\sigma(\sigma+\delta)+\sin 2k_\sigma\sigma} \quad (1)$$

The dimensions h, δ, and σ are indicated in Figure 1 and $\Delta\epsilon=\epsilon_f-\epsilon_d$, is the difference of relative dielectric constants between the ferrite and dielectric insert. Then the transcendental equation for the fundamental TE_{10} (or LSE_{10}) mode is given by:

$$k_{\sigma o}\tan k_{\sigma o}(\delta+\sigma)=k_{ao}\cot k_{ao}a$$
$$k_{\sigma o}^2+k_{ao}^2=\frac{\omega^2}{c^2}(\epsilon^{eff}-1) \quad (2)$$

These equations are solved for k_{ao}, $k_{\sigma o}$ and hence $\beta^2=k_{ao}^2+\omega^2/c^2$. Using these values in the perturbation formula the appropriate functions are evaluated over the magnetized toroidal ferrite window frame configuration. Only the vertically polarized portions contribute to the differential phase shift $\Delta\beta=\beta_+-\beta_-$ for the opposite directions of propagation or reversal of magnetization. $\Delta\beta$ is given by:

$$\Delta\beta=2\chi_{xy}\frac{h-2\delta}{h}\left[\frac{\cos 2k_{\sigma o}\sigma-\cos 2k_{\sigma o}(\sigma+\delta)+\frac{\sin k_{\sigma o}(\sigma+\delta)}{k_{\sigma o}h}}{(\sigma+\delta)+\frac{\sin 2k_{\sigma o}(\sigma+\delta)}{2k_{\sigma o}}+\frac{k_{\sigma o}\sin 2k_{\sigma o}(\sigma+\delta)}{k_{ao}\sin 2k_{ao}a}\left(\frac{\sin 2k_{ao}a}{2k_{ao}}-a\right)}\right] \quad (3)$$

A similar procedure for the double toroid separated by a dielectric slab yields an explicit expression of comparable complexity in terms of the pertinent parameters. Both expressions allow more flexibility and ease of computation than the ersatz idealized triple slab transcendental equation. The comparison between the perturbation and exact equivalent solutions with experiment shows the former to be superior as indicated in Figure 2 over a broadband of frequencies. Figure 3 shows comparison of our theory for another single toroid and a double toroid over comparable bandwidth with good results.

HIGHER ORDER MODES

Higher order modes can be excited when their cut-off frequencies occur in the operating band. Inhomogeneities and off-diagonal tensor components can couple these to the

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dominant mode. Taking a linear combination of these modes and applying the perturbation formula to each mode as the dominant mode leads to a secular equation which can be solved for the propagation constant of all the perturbed modes. The secular equation contains matrix elements in which the off-diagonal components indicate the degree of coupling between the various waveguide modes. These are integrals evaluated in terms of the orthonormal functions and the dielectric and magnetic perturbations as before but between separate modes. From the solution of the secular equation the relative amplitudes of each mode can be determined. The secular equation can be expanded about each mode to second order neglecting higher order products of the matrix elements to yield a value β_i where i is the dominant mode corrected for the coupling to other modes

$$\beta_i=\beta_{oi}+M_{ii}+\sum_{j\neq i}\frac{M_{ij}^2}{\beta_i-\beta_j} \quad (4)$$

The simplest example of the procedure is to consider a square waveguide with a longitudinally magnetized ferrite toroid surrounding an electro-optical crystal used as a magnetically tunable modulator (4). Our procedure yields the propagation constant for the two counter rotating modes as follows:

$$\beta\pm=\overline{\beta}(1+0.3\,\chi_{xx}^{eff}\pm0.2\,\chi_{xy}^{eff}) \quad (5)$$

where the numerical factors are for specific geometrical dimensions, the effective susceptibilities in terms of demagnetizing factors and $\overline{\beta}$ for an equivalent completely filled square guide with an effective dielectric constant evaluated from the perturbation formula as in Equation (1). Details are presented in reference (4).

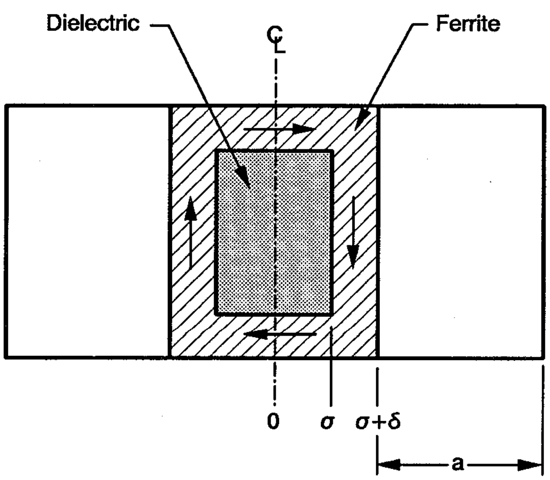

Figure 1
Toroidal ferrite tube with dielectric insert in rectangular waveguide. Magnetization indicated by arrows.

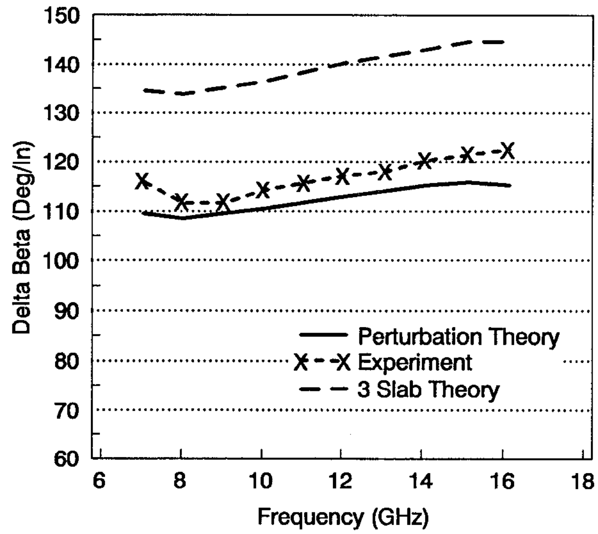

Figure 2
Comparison of theory, experiment and 3 slab approximation for single toroidal phase shifter.

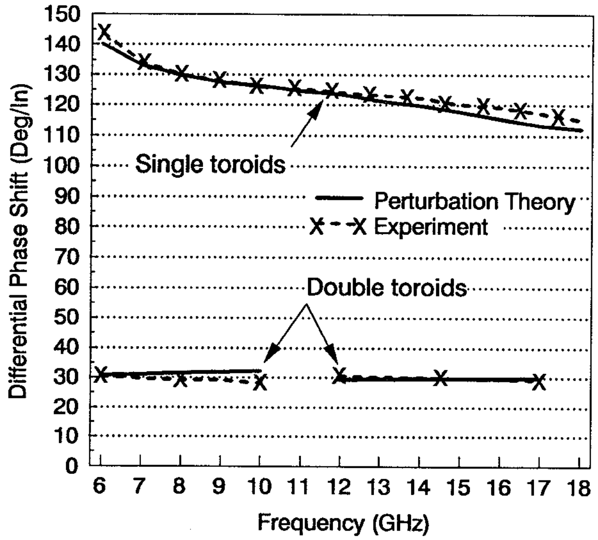

Figure 3
Comparsion of theory and experiment for single and double broadband phase shifters.

CONCLUSIONS

The use of the perturbation procedure represents a new technique for solving problems in an inhomogeneously loaded waveguide (or cavity) for which exact solutions are not possible. The results yield quite accurate values when the geometry and associated depolarizing and demagnetizing factors are properly accounted for. Since the fields are evaluated even to higher order when the coupling of modes are calculated, the impedance of the waveguide can be obtained from the line integrals of equivalent voltages and currents from the total electric and magnetic fields using the voltage-current relations or from the Poynting vector.

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